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TO FIND THE RESULTS OF THE DOMINATION NUMBER AND THE DIAMETER OF CIRCULAR-ARC GRAPHS

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Abstract
Among the various applications of the theory of domination and the distance, the most often discussed is a communication network. This network consists of communication links all distance between affixed set of sites. Circular-arc graphs are rich in combinatorial structures and have found applications in several disciplines such as Biology, Ecology, Psychology, Traffic control, Genetics, Computer sciences and particularly useful in cyclic scheduling and computer storage allocation problems etc. Then the problem is what is the fewest number of communication links such that at least one additional transmitter would be required in order that communication with all sites as possible. In this paper the comparison of the eccentricity and the diameter of circular - arc graph.

Key Words: Applications, Domination, Communication, Comparison.

Introduction

Circular - arc graphs are rich in combinatorial structures and have found applications in several disciplines such as Biology, Ecology, Genetics, Computer Science and particularly useful in cyclic scheduling.

Consider A = {A_1, A_2, ……, A_n}, a family of arcs on a circle C. Each endpoint of the arc A_i is assigned a positive integer called a co – ordinate. The endpoints are located at the circumference of C in ascending order of the values of the co – ordinates in the clockwise direction.

Suppose that an arc begins at c and ends at the point d in the clockwise direction. Then we denote such an arc by [c, d] and the points c and d arc called respectively, the head point and tail point of the arc. The arcs are given labels in the increasing order of their head points.

If the head point of an arc is less than the tail point of the arc, then the arc is called a forward arc. Otherwise it is called a backward arc. A is called a proper arc family if no arc in A contains another arc.

Thus, the arc family A is now denoted by A = {1, 2, ….., n} and let G be its corresponding circular – arc graph..

In this paper, we consider the bondage number *b(G)* for a circular - arc family *A* and *G* is a circular - arc graph corresponding to a circular - arc family *A*, which is defined as the minimum number of edges whose removal results in a new graph with larger domination number, A subset D of v is said to be a dominating set of G if every vertex not in D is adjacent to vertex in D.The domination number γ(G) of a graph G is the minimum cardinality of a dominating set in G[1].

Where δ(x, y) denotes the length of shortest path joining the vertices x and y . The average distance can be used as a tool in analytic networks where the performance time is proportions to the distance between any two nodes. It is a measure of the time needed in the average case as opposed to the diameter, which indicates the maximum performance time and also the formulated of the walk, length of a walk, eccentricity, radius and diameter of the graph.

A walk from v_0 to v_n is an alternating sequence W= $\{v_{0,} e_1,v_1,e_2,\ldots..v_{n-1},e_n,v_n\}$ of vertices and edges such that end points $(e_i)=\{v_{i-1},v_i\}$; for i={1,2,…n}.

In a simple graph, there is only one edge between two consecutive vertices of a walk, so one could abbreviate the walk as $W=\{v_{0,} e_1,v_1,e_2,\ldots..v_{n-1},e_n,v_n\}$.

The length of a walk or directed walk is the number of edge steps in the walk sequence. A walk of length zero. i.e with one vertex and no edges is called a trivial walk.

Main Theorems
Theorem.1
Let $\gamma_{ns}(D)$ be a domination number of the given circular-arc graph. If a_i, a_j are any two arcs in A such that $a_j$is contained in a_i and $a_i \in D$ and if p is an arc in A, which is to the left of $a_j$in clock wise direction, such that $p<a_j$ and p intersects a_j and if there is at least one $a_k>a_j$, such that a_k intersects a_j, then the non-split domination number $\gamma_{ns}(D)$ is greater than diameter of G.
i.e.$\gamma_{ns}(D) > Diam(G)$

Proof:Let $\{a_1, a_2, \ldots.., a_n\}$be a family of n-arcs on a circle A and let G be a circular-arc graph corresponding to circular-arc family I. First we will prove that $\gamma_{ns}(D)$ is a minimal domination number og G. Then for every vertex v in $\gamma_{ns}(D)$, $\gamma_{ns}(D) - \{v\}$ is not a domination number .Thus some vertex u in $V - \gamma_{ns}(D) \cup \{v\}$ is not dominated by any vertex in $\gamma_{ns}(D) - \{v\}$. Now either u = v or $u \in V - \gamma_{ns}(D)$. If u = v, then v is an isolated vertex of $\gamma_{ns}(D)$. If $u \in V - \gamma_{ns}(D)$ and u is not dominated by $\gamma_{ns}(D) - \{v\}$, but is dominated by $\gamma_{ns}(D)$, then u is adjacent to any two vertices v in $\gamma_{ns}(D)$, that is $N(u) \cap \gamma_{ns}(D) = \{v\}$.

Next we will find the non-split domination number of G. Suppose there is atleast one arc $k \neq i$, k > j such that k intersects j. Then it is obvious that in $<V - \gamma_{ns}(D)>$ such that p intersects j. Further by hypothesis there is atleast one p < j in $<V - \gamma_{ns}(D)>$ such that p intersect j. Hence j is connected to its left as well as to its right, so that there will not be any disconnection in $<V - \gamma_{ns}(D)>$. Again we will prove that let G be a circular-arc graph corresponding to circular-arc family A= $\{a_1, a_2, \ldots.., a_n\}$ be a circular-arc family on a circle. Where each $a_i$is an arc. Without lot of generality assume that the end points of all arcs are distinct and no arcs covers the entire circle.

Where $\delta(a_i, a_j)$ denotes the length of a distance joining the vertices ai and a_j. The average distance can be used as a tool ina analytical network where the performance time is proportional to the distance between two vertices (a_i, a_j). Now we will prove that the diameter of G. in this connection first we prove that the eccentricity $e(a_i)$ of a vertex a_i to a vertex furthest from a_i. Vertex a_j is said to be a furthest neighbor of the vertex a_i of $\delta(a_i, a_j) = (a_i)$. The diameter of a graph Gis the minimum among all eccentricities. The eccentricity $e(a_i) = \{ \delta(a_i, a_j) : a_j \in V \}$. Infact we have to find the diameter of G. Since $Diam(G) = max \{ e(a_i) : a_j \in V \}$. By the theorem our proportion to find $\gamma_{ns}(D) > Diam(G)$.
Therefore the theorem is hold.

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Illustration

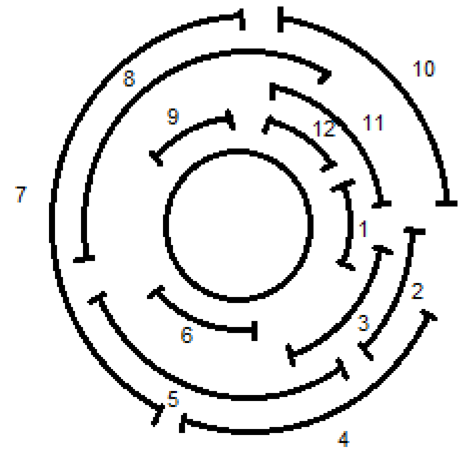

Fig. 2: Circular-arc family A

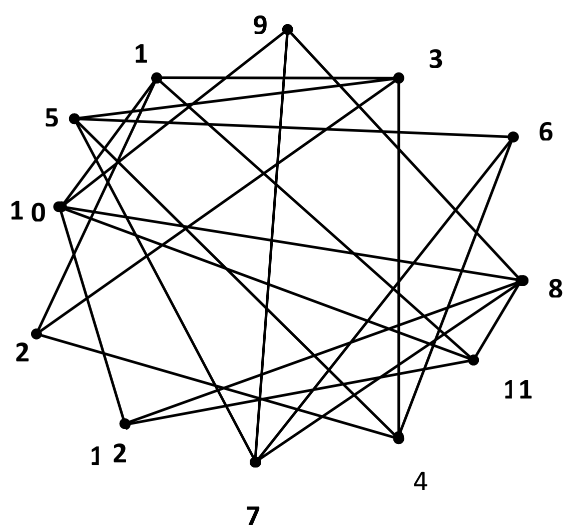

Fig. 2: Circular-arc graph G

Dominating Set = {1,4,8}, $\gamma_{ns}(G)=3$

To find the distances from

d(7,1)=3	d(8,1)=2	d(9,1)=2	d(10,1)=1	d(11,1)=1	d(12,1)=2
d(7,2)=3	d(8,2)=3	d(9,2)=3	d(10,2)=2	d(11,2)=2	d(12,2)=3
d(7,3)=2	d(8,3)=3	d(9,3)=3	d(10,3)=2	d(11,3)=2	d(12,3)=3
d(7,4)=2	d(8,4)=3	d(9,4)=3	d(10,4)=3	d(11,4)=3	d(12,4)=4
d(7,5)=1	d(8,5)=2	d(9,5)=2	d(10,5)=3	d(11,5)=3	d(12,5)=3
d(7,6)=1	d(8,6)=2	d(9,6)=2	d(10,6)=3	d(11,6)=3	d(21,6)=3
d(7,7)=0	d(8,7)=1	d(9,7)=1	d(10,7)=2	d(11,7)=2	d(12,7)=2
d(7,8)=1	d(8,8)=0	d(9,8)=1	d(10,8)=1	d(11,8)=1	d(12,8)=1
d(7,9)=1	d(8,9)=1	d(9,9)=0	d(10,9)=1	d(11,9)=2	d(12,9)=2
d(7,10)=2	d(8,10)=1	d(9,10)=1	d(10,10)=0	d(11,10)=1	d(12,10)=1
d(7,11)=2	d(8,11)=1	d(9,11)=2	d(10,11)=1	d(11,11)=0	d(12,11)=1
d(7,12)=2	d(8,12)=1	d(9,12)=2	d(10,12)=1	d(11,12)=1	d(12,12)=0

e(1) = max {0, 1, 1, 2, 2, 3, 3, 2, 2, 1, 1, 2}=3
e(2) = max {1, 0, 1, 1, 2, 2, 3, 3, 3, 2, 2, 3}=3
e(3) = max {1, 1, 0, 1, 1, 2, 2, 3, 3, 2, 2, 3}=3
e(4) = max {2, 1, 1, 0, 1, 1, 2, 3, 3, 3, 3, 4}=4
e(5) = max {2, 2, 1, 1, 0, 1, 1, 2, 2, 3, 3, 3}=3
e(6) = max {3, 2, 2, 1, 1, 0, 1, 2, 2, 3, 3, 3}=3
e(7) = max {3, 3, 2, 2, 1, 1, 0, 1, 1, 2, 2, 2}=3
e(8) = max {2, 3, 3, 3, 2, 2, 1, 0, 1, 1, 1, 1}=3
e(9) = max {2, 3, 3, 3, 2, 2, 1, 1, 0, 1, 2, 2}=3
e(10) = max {1, 2, 2, 3, 3, 3, 2, 1, 1, 0, 1, 1}=3
e(11) = max {1, 2, 2, 3, 3, 3, 2, 1, 2, 1, 0, 1}=4
e(12) = max {2, 3, 3, 4, 3, 3, 2, 1, 2, 1, 1, 0}=4
e(v) = max {3, 3, 3, 4, 3, 3, 3, 3, 3, 3, 3, 4}=4

Diameter

Diam(G) = max {3, 3, 3, 4, 3, 3, 3, 3, 3, 3, 3, 4}=4

Therefore Diam(G) = 4

Therefore Diam(G) >γ_{ns}(D)

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Theorem: Let A= {a_1, a_2, ….., a_n} be a circular-arc family and let G be a circular-arc graph corresponding to circular-arc family. If a_j = 2 is the arc contained in a_i = 1 and $a_i \in \gamma_{ns}(D)$ and there is atleast one arc that intersect a_j to its right other than a_i. Then the non-split domination number is greater than the diameter of G.

$\gamma_{ns}(D) > Diam(G)$

Proof

In this theorem we have already proved the domination number of G in theorem.1. Now we have to find the non-split domination number $\gamma_{ns}(D)$. From Theorem let a_j = 2 be the arc a_i = 1 and $a_i \in \gamma_{ns}(D)$. Suppose a_k is any arc, $a_k \neq a_j$,$a_k > a_j$ such that a_k intersects a_j.

Then <V - $\gamma_{ns}(D)$> does not contain a_i. But the induced subgraph <V - $\gamma_{ns}(D)$>, a_j is connected to a_k, so that there is no disconnection for a_j to its right in clock wise direction. Then we have a connected graph G. Next we will find the average distance of G as well as the diameter already we proved in theorem.1.

Illustration

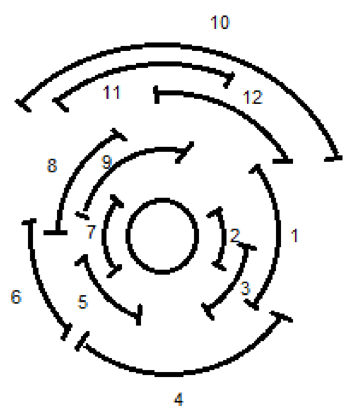

Fig. 3: Circular-arc family A

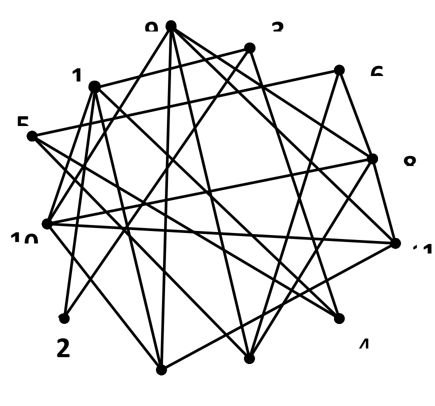

Fig. 4: Circular-arc graph G

Dominating Set = {4,7,9}, $\gamma_{ns}(G)$=3

To find the distances from G

d(1,1)=0	d(2,1)=1	d(3,1)=1	d(4,1)=1	d(5,1)=2	d(6,1)=3
d(1,2)=1	d(2,2)=0	d(3,2)=1	d(4,2)=2	d(5,2)=3	d(6,2)=4
d(1,3)=1	d(2,3)=1	d(3,3)=0	d(4,3)=1	d(5,3)=2	d(6,3)=3
d(7,1)=3	d(8,1)=2	d(9,1)=2	d(10,1)=1	d(11,1)=2	d(12,1)=1
d(7,2)=4	d(8,2)=3	d(9,2)=3	d(10,2)=2	d(11,2)=3	d(12,2)=2
d(7,3)=3	d(8,3)=3	d(9,3)=3	d(10,3)=2	d(11,3)=3	d(12,3)=2
d(7,4)=2	d(8,4)=3	d(9,4)=3	d(10,4)=2	d(11,4)=3	d(12,4)=2
d(7,5)=1	d(8,5)=2	d(9,5)=2	d(10,5)=3	d(11,5)=3	d(12,5)=3
d(7,6)=1	d(8,6)=2	d(9,6)=2	d(10,6)=2	d(11,6)=2	d(21,6)=3
d(7,7)=0	d(8,7)=1	d(9,7)=1	d(10,7)=2	d(11,7)=2	d(12,7)=2
d(7,8)=1	d(8,8)=0	d(9,8)=1	d(10,8)=1	d(11,8)=1	d(12,8)=2
d(7,9)=1	d(8,9)=1	d(9,9)=0	d(10,9)=1	d(11,9)=1	d(12,9)=1
d(7,10)=2	d(8,10)=1	d(9,10)=1	d(10,10)=0	d(11,10)=1	d(12,10)=1
d(7,11)=2	d(8,11)=1	d(9,11)=1	d(10,11)=1	d(11,11)=0	d(12,11)=1
d(7,12)=2	d(8,12)=1	d(9,12)=1	d(10,12)=1	d(11,12)=1	d(12,12)=0

Eccentricity

e(1) = max {0, 1, 1, 1, 2, 3, 3, 2, 2, 1, 2, 1}=3
e(2) = max {1, 0, 1, 2, 3, 4, 4, 3, 3, 2, 3, 2}=4
e(3) = max {1, 1, 0, 1, 2, 3, 3, 3, 3, 2, 3, 2}=3
e(4) = max {1, 2, 1, 0, 1, 2, 2, 3, 3, 2, 3, 2}=3
e(5) = max {2, 3, 2, 1, 0, 1, 1, 2, 2, 3, 3, 3}=3
e(6) = max {3, 4, 3, 2, 1, 0, 1, 1, 2, 2, 2, 3}=4
e(7) = max {3, 4, 3, 2, 1, 1, 0, 1, 1, 2, 2, 2}=4
e(8) = max {2, 3, 3, 3, 2, 1, 1, 0, 1, 1, 1, 2}=3
e(9) = max {2, 3, 3, 3, 2, 2, 1, 1, 0, 1, 1, 1}=3
e(10) = max {1, 2, 2, 2, 3, 2, 2, 1, 1, 0, 1, 1}=3
e(11) = max {2, 3, 3, 3, 3, 2, 2, 1, 1, 1, 0, 1}=3
e(12) = max {1, 2, 2, 2, 3, 3, 2, 2, 1, 1, 1, 0}=3
e(v) = max {3, 4, 3, 3, 3, 4, 4, 3, 3, 3, 3, 3}=4

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Diameter

Diam(G) = max {3, 4, 3, 3, 3, 4, 4, 3, 3, 3, 3, 3}=4

Therefore Diam(G) = 4

Therefore Diam(G) >γ_{ns}(D)

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